Quantum Information Processing Conference
RIGA 2026

Symmetric localizable multiparty quantum measurements

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Joint work with Cyril Branciard, Alex Pozas, Nicolas Gisin

arXiv: 2509.01851, 2509.01852

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Entanglement in quantum measurements

Entanglement

- EPR
- Schmidt (partial entanglement)
- Werner (mixed states and noise)
- GHZ, W,… (multipartite)

Joint measurements

- Bell
- GHZ
- Partially entangled (???)
- High dim, multiparty,…. (???)

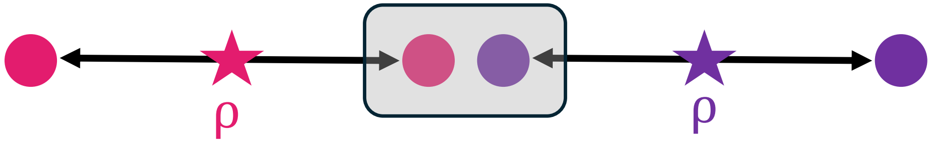
We lack canonical examples

Joint measurements in correlations

Entanglement properties of **states**

Correlations between **subsystems** of a single source

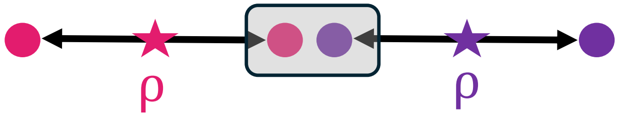
Entanglement in **measurements**



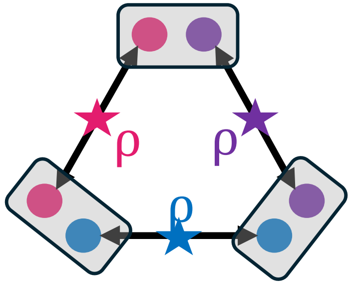
Correlations between different **independent sources**

Examples: EA PM, MDI quantum cryptography, networks,…

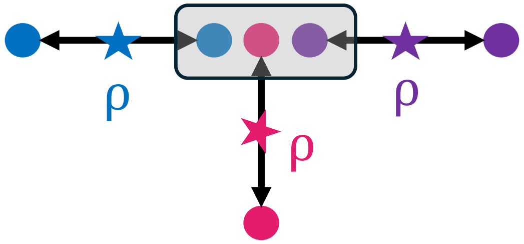
Network nonlocality



Bell state measurement:
Entanglement swapping

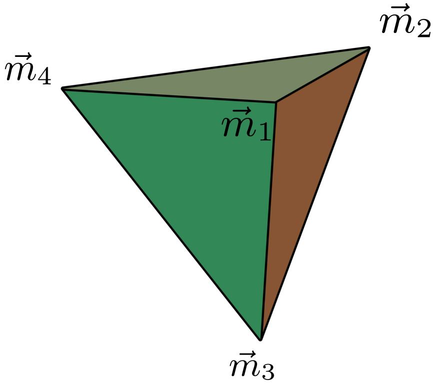


Elegant joint measurement:
Genuine network nonlocality
Twisted Bell state measurement:
Randomness without inputs



GHZ measurement:
Conference key agreement

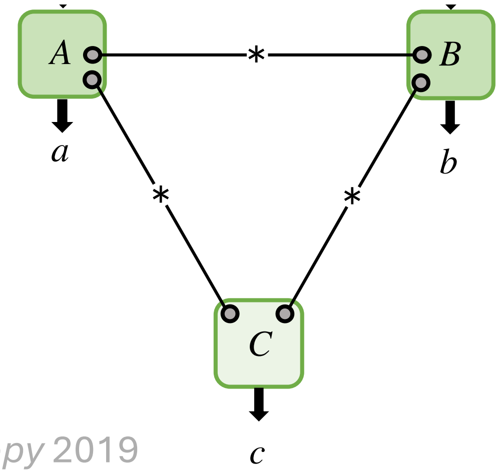
The Elegant Joint Measurement (EJM)



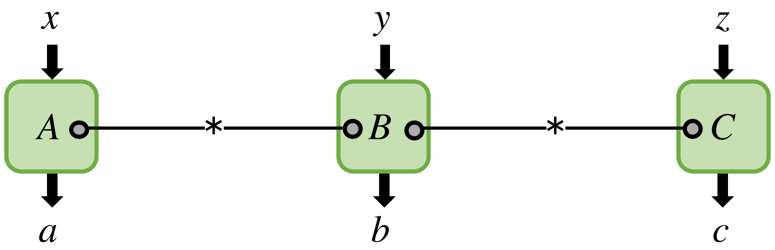
$$|\psi_{\mathrm{EJM},i}\rangle = \frac{\sqrt{3}+1}{2\sqrt{2}}\,|\vec{m}_i, -\vec{m}_i\rangle + \frac{\sqrt{3}-1}{2\sqrt{2}}\,|-\vec{m}_i, \vec{m}_i\rangle\ ,$$

Benchmark example for network correlations with non-maximally entangled measurements

Triangle



Bilocality



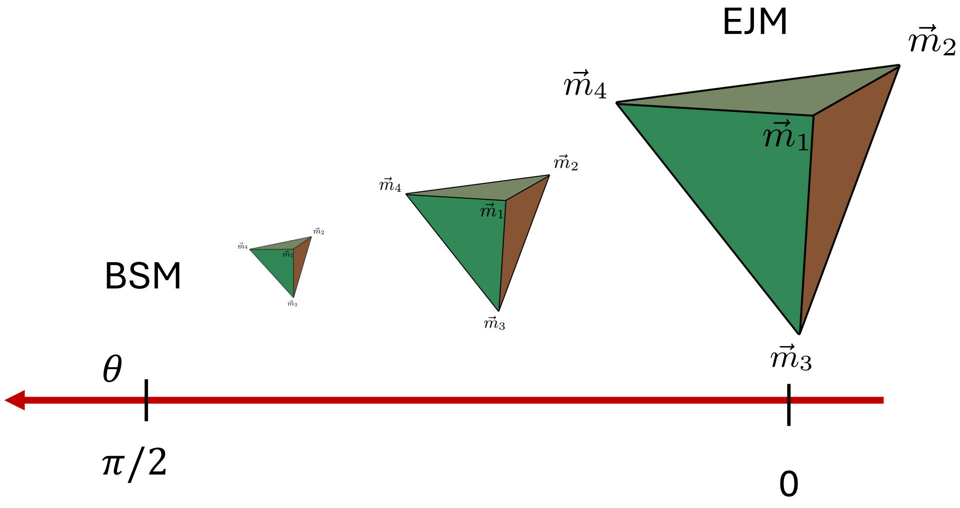
N. Gisin, Entropy 2019
V. Gitton, R. Renner 2025

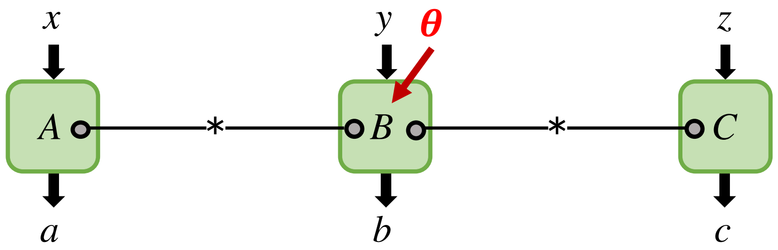
A. Tavakoli, C. Branciard, N. Gisin, PRL (2021)

The Elegant Joint Measurement (EJM)

$$|\psi_{\mathrm{EJM},i}\rangle = \frac{\sqrt{3} + \boldsymbol{e^{i\theta}}}{2\sqrt{2}} |\vec{m}_i, -\vec{m}_i\rangle + \frac{\sqrt{3} - \boldsymbol{e^{i\theta}}}{2\sqrt{2}} |-\vec{m}_i, \vec{m}_i\rangle \ ,$$

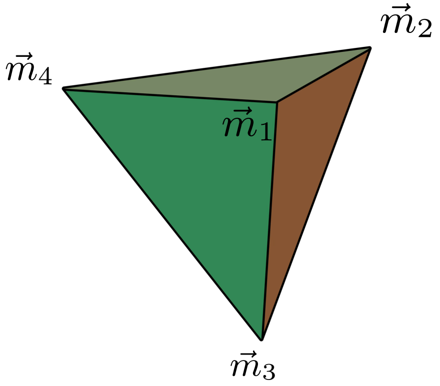
Structured family of measurements: How does degree of entanglement affect network correlations?





A. Tavakoli, C. Branciard, N. Gisin, PRL (2021)

Properties of the EJM



$$|\psi_{\mathrm{EJM},i}\rangle = \frac{\sqrt{3}+1}{2\sqrt{2}}\,|\vec{m}_i, -\vec{m}_i\rangle + \frac{\sqrt{3}-1}{2\sqrt{2}}\,|-\vec{m}_i, \vec{m}_i\rangle\ ,$$

Iso-entangled → **Local encodability**

(generated from a single fiducial state)

Local tetrahedral symmetry → **Pauli covariance**

$$G^{(2)}_{\mathrm{tetra}} \equiv \langle X\otimes X,\ Z\otimes Z\rangle$$

➔ EJM as a group orbit basis $\left\{ \underbrace{U_g\,|\psi\rangle} \;\middle|\; U_g \in G^{(2)}_{\mathrm{tetra}} \right\}$

EJM fiducial state

Generalizing EJM symmetry group

Desiderata: (i) globally abelian
(ii) locally contains all the Pauli’s (stabilizes the tetrahedron)

$$G^{(n)}_{\text{tetra}} = \left\langle Z^{(1)}Z^{(2)}, Z^{(2)}Z^{(3)}, \dots, Z^{(n-1)}Z^{(n)}, X^{\otimes n} \right\rangle \cong \mathbb{Z}_2^n,$$

➔Define tetrahedral ONBs

$$\mathcal{B} = \left\{ U_g \left|\psi\right\rangle \;\middle|\; U_g \in G^{(n)}_{\text{tetra}} \right\}$$

GHZ states

➔Admissible fiducials

$$\left|\psi^{(n)}_{\text{tetra}}\right\rangle = \frac{1}{\sqrt{2^n}} \sum_{(\vec{z},x)\in\{0,1\}^n} e^{i\alpha_{\vec{z},x}} \left|\Phi_{\vec{z},x}\right\rangle$$

$2^n - 1$ parameters

Large family of bases, local tetrahedral geometry determined (non-linearly) by $\boldsymbol{\alpha}$

Localizability

Given an eigenstate ψ_i, recover the classical label i

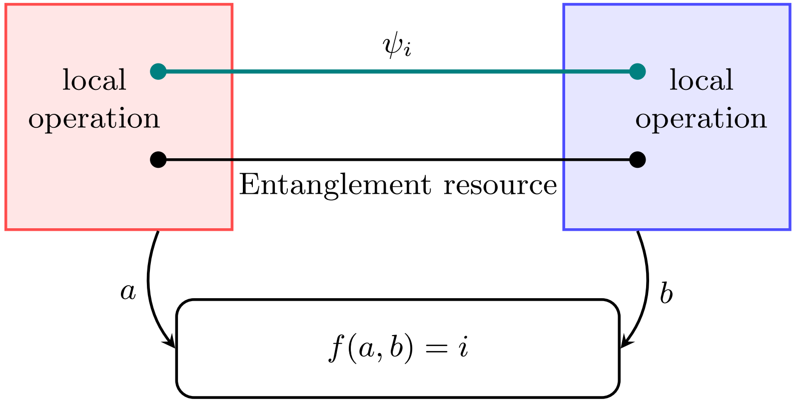
= Local **decodability** ⟷ local encodeablity

Not possible (perfectly) without resources

Entanglement cost =

Measure of nonlocal complexity

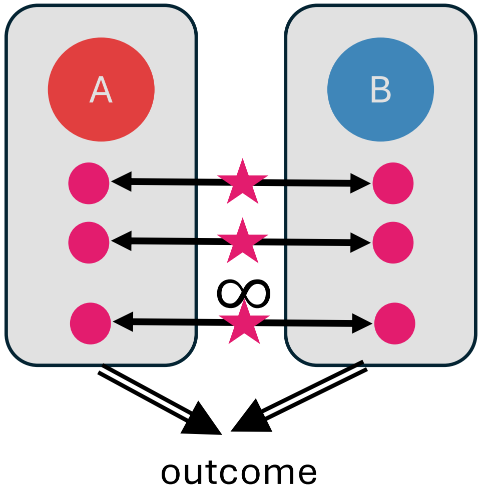
→ Classify measurements accordingly



JP, A. Pozas, F. Del Santo, N. Gisin, PRX (2025)

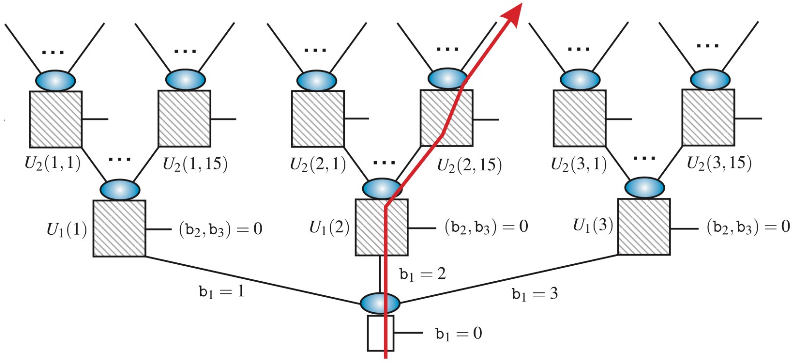
Vaidman protocol

Groisman & Reznik, PRA (2002)
Vaidman, PRL (2003)



- Universal but requires ∞ entanglement
- After finite number of **steps $\boldsymbol{k}$** : subset of measurements with low complexity
- Algebraic characterization: $M = \sum_g |\psi_g\rangle \langle g|$

Membership in **hierarchy of sets** $\mathcal{V}_1 \subsetneq \cdots \subsetneq \mathcal{V}_{k-1} \subsetneq \mathcal{V}_k \subsetneq \cdot\cdot$



EJM + 4 others

BSM, twisted

Product bases

In general difficult to characterize

Relation to the Clifford hierarchy

Recursive algebraic hierarchy organizing all unitaries by their action on Paulis

$$\mathcal{C}_k := \left\{ U \in \mathcal{U}(2^n) \,\middle|\, UPU^\dagger \in \mathcal{C}_{k-1} \;\; \forall\, P \in \mathcal{P}_n \right\} \qquad \mathcal{C}_1 := \mathcal{P}_n$$

Originally defined (Gottesman & Chuang, 1999) in context of fault tolerant computation

Can be understood as **complexity of nonlocal computation**

$$\boxed{\mathcal{C}_k \;\subsetneq\; \mathcal{V}_k}$$

Intuition: Measurements are a subset of all computations

Localizability of Tetrahedral bases

Structure of tetrahedral bases → significant simplification

Normal form

$$\left|\psi^{(n)}_{\text{tetra}}\right\rangle = \frac{1}{\sqrt{2^n}} \sum_{(\vec{z},x)\in\{0,1\}^n} e^{i\alpha_{\vec{z},x}} \left|\Phi_{\vec{z},x}\right\rangle$$

$$|\psi\rangle = V\,|0\rangle^{\otimes n} \qquad V = S(n)\,H_n\,D_{\vec{\alpha}}\,H^{\otimes n}$$

Theorem: Localizability of tetrahedral bases determined by diagonal phase matrix

$$M_\psi \;:=\; \sum_{g:U_g\in G^{(n)}_{\text{tetra}}} U_g\;|\psi\rangle\,\langle g|\,,$$

$$\boxed{D_{\vec{\alpha}} \in \mathcal{C}_k \quad \Longrightarrow \quad M_\psi \in \mathcal{C}_k \subsetneq \mathcal{V}_k}$$

JP, C. Branciard, A. Pozas, N. Gisin, arXiv (2025)

Diagonal gates in the Clifford hierarchy

Diagonal unitaries can be approximated as $D_{f_m} = \sum_{\vec{z} \in \mathbb{Z}_2^n} \exp\left[i \, \frac{2\pi}{2^m} \, f_m(\vec{z})\right] \, |\vec{z}\rangle\langle\vec{z}|$

where $f_m : \mathbb{Z}_2^n \to \mathbb{Z}_{2^m}$

$$f_m(\vec{z}) = \sum_{S \subseteq [n] \;\; S \neq \emptyset} a_S \prod_{i \in S} z_i \mod 2^m$$

The Clifford level k of these unitaries is determined by **degree $|S|$** and **precision m** of the polynomial

$$k \;=\; \max_{S \neq \emptyset :\, a_S \neq 0} \left[(m - \nu_2(a_S) - 1) + |S|\right]$$

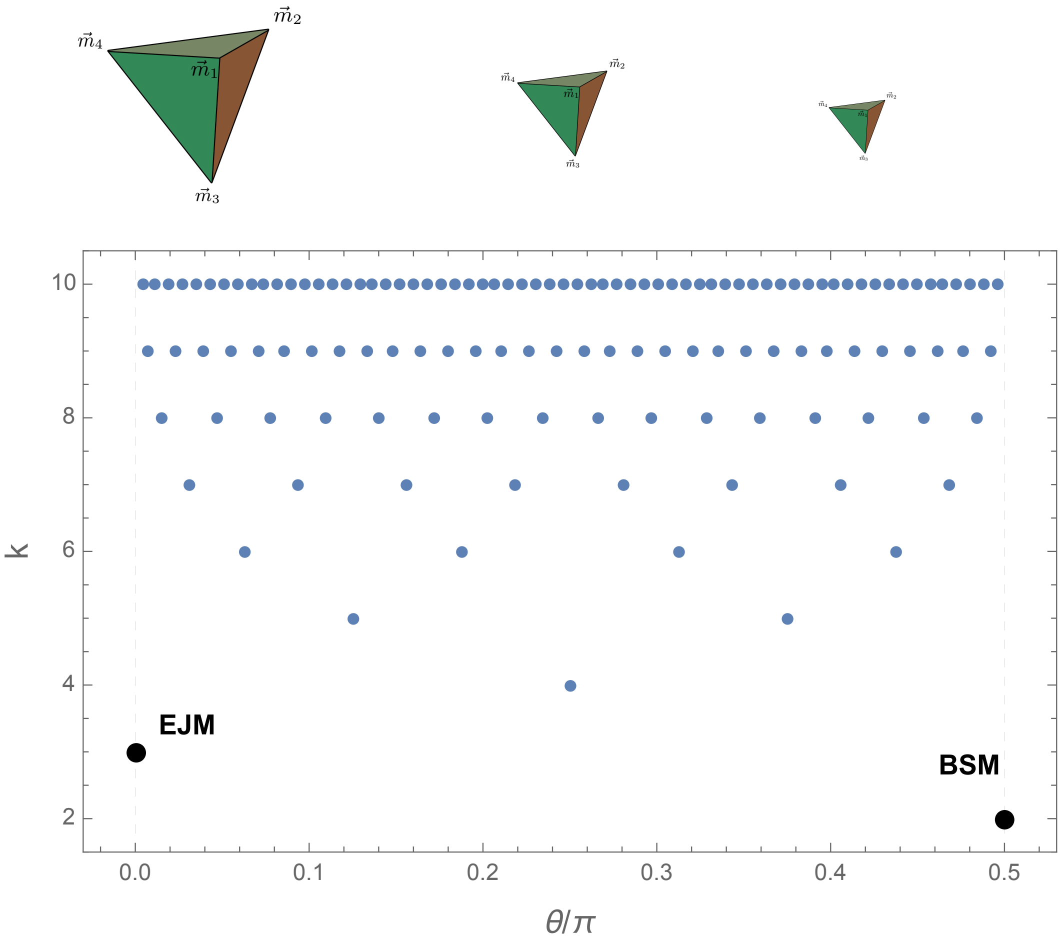
S. X. Cui, D. Gottesman, and A. Krishna, PRA (2017)

The EJM family

All tetrahedral bases with fiducial state normal form

$$|\psi_\theta\rangle = S H_2 D(\theta) H^{\otimes 2} |00\rangle$$

Compute level



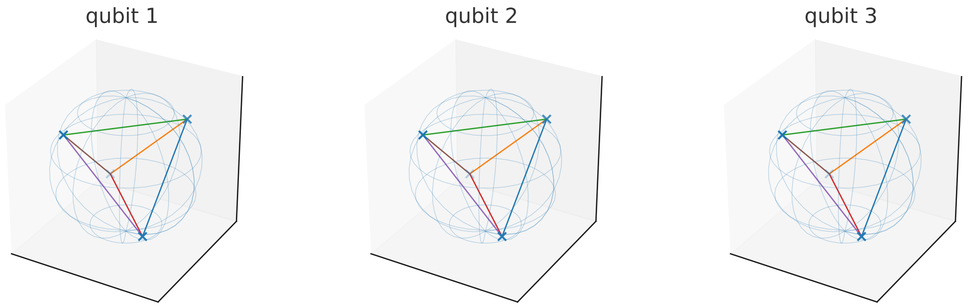
Using **localizability** to find a ‘**multiqubit EJM**’ among tetrahedral bases

1) Iterate over all low-level polynomials

$$
\begin{aligned}
f_m(z_1, z_2, z_3) = \; & a_1\, z_1 + a_2\, z_2 + a_3\, z_3 \\
& + a_{12}\, z_1\, z_2 + a_{13}\, z_1\, z_3 + a_{23}\, z_2\, z_3 \\
& + a_{123}\, z_1\, z_2\, z_3 \quad \mod 2^m .
\end{aligned}
$$

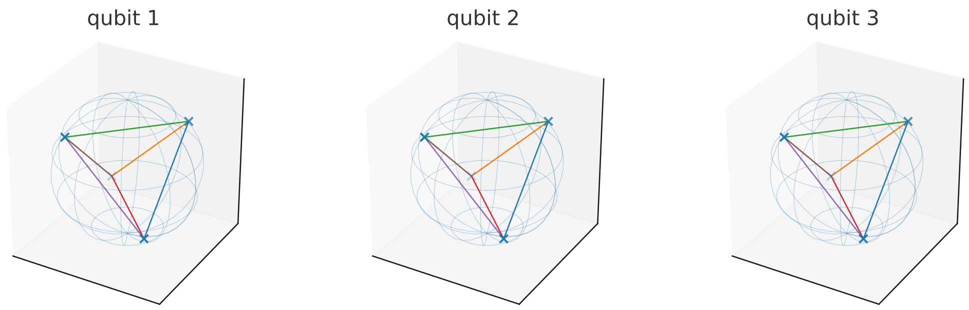
→ Finitely many options at each level

2) Look for isotropic Bloch patterns



JP, N. Gisin, arXiv (2025)

The 3-qubit EJM



At level $k = 4$ **unique isotropic geometry**:

Regular tetrahedron precisely ½ volume of the 2 qubit EJM

Different entanglement types
(all equal pairwise concurrence,
different 3-tangle$>$ 0)

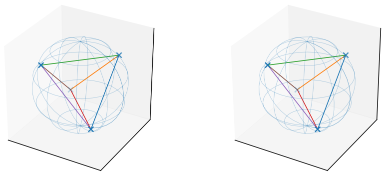
$$f(z_1, z_2, z_3) = 3z_1z_3 + z_2z_3 + 3z_1z_2z_3$$

$$\begin{aligned}|\psi\rangle = c_0 \; &|-\vec{m}_1, -\vec{m}_1, -\vec{m}_1\rangle \\ &+ c_1 \left(|\vec{m}_1, -\vec{m}_1, -\vec{m}_1\rangle + \text{perms.}\right) \\ &+ c_2 \left(|\vec{m}_1, \vec{m}_1, -\vec{m}_1\rangle + \text{perms.}\right) \\ &+ c_3 \; |\vec{m}_1, \vec{m}_1, \vec{m}_1\rangle \; .\end{aligned}$$

JP, N. Gisin, arXiv (2025)

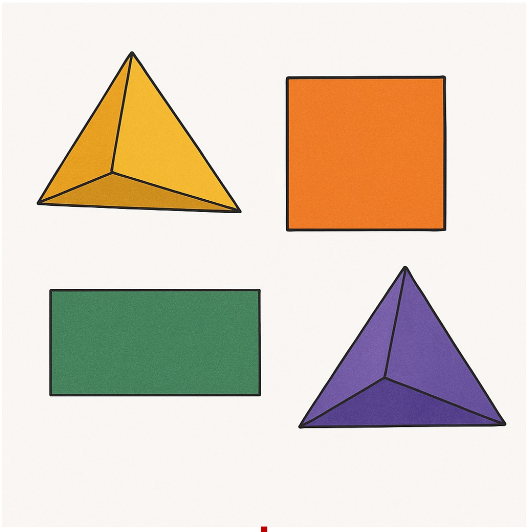
Summary

Two qubit EJM



Local encodability
Local tetrahedral symmetry
Low localization cost

Family of tetrahedral measurements



Localizability
problem easier

Low localization cost

Three qubit EJM